ANNEE SPECIALE SUR LES PHENOMENES NON LINEAIRES

1987 - 1988

sous le haut patronage du C.N.R.S. , du Ministère des Universités et de la Recherche, avec le concours de la S.M.A.I., de la S.M.F. et de la D.R.E.T

EQUATIONS AUX DERIVEES PARTIELLES ET MODELES CONTINUS DE TRANSITIONS DE PHASES

Nice, 18-22 Janvier 1988

Séminaire conjoint C.N.R.S. - N.S.F.
sous les auspices de l'International Society for the Interactions of Mechanics and Mathematics

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Michel RASCLE, Denis SERRE, Marshall SLEMROD

Comité Scientifique :
John BALL, James GLIMM, Morton GURTIN, Maurice KLEMAN,Gérard MAUGIN.

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Location :
Hôtel Nikaiä, 39 rue Clément Roassal, 06 000 Nice,Tél. 93 87 59 59, **except** on the afternoons of Tuesday 19 th and Friday 22 th (Hôtel Plaza, 12, Avenue de Verdun, Nice) and of Thursday 21rst (Amphithéatre de Physique 1, Université de Nice,Parc Valrose, 06 034 Nice Cedex,Tél. 93 52 98 30).

YOUNG'S MEASURES AND THE MICRO-STRUCTURE OF CRYSTALS (I, II)

John BALL, R.[illegible] AMES(*)

We discuss the rôle of the Young measure in predicting the microstructure of crystals, and the related problem of characterizing the Young measures corresponding to a sequence of gradients and having support in a given set of matrices. The underlying idea is that the stored-energy functions for crystals are in general non-elliptic, so that minimizing sequences typically have finer and finer structure. A new and more satisfactory basis is thereby is given for the classical crystallographic theory of Martensite due to Wechsler, Lieberman and Read (1953) . Plans for future theoretical and experimental work are discussed.

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A TWO PHASE FLOW MODEL SUITABLE FOR PRESSURIZED WATER REACTOR SAFETY STUDIES: THE CATHARE CODE

D. BESTION
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Abstract

In the frame of pressurized water reactor safety studies , the CATHARE code is elaborated to simulate steam water two-phase flows in accidental situations. A 1-dimension 2-fluids 6-equations model is chosen for all flows in pipes.

A mass , momentum and energy balance equation is written for each fluid . They are derived from local instantaneous equations by using time and space averaging . Transverse pressure gradient effects can be correctly taken into account in the case of a stratified flow by adding a differential term in the momentum equations. For other flow patterns , this term is written so as to provide the hyperbolicity of the system of equations ; the well-posedness of the problem is then ensured. The discretization principles used in the finite difference scheme are based on the staggered mesh and the donor cell technics . The time discretization is a fully implicit scheme .

The code has a modularity that allows the representation of a multi component circuit . The resolution of the linearized system of equations is a 3 steps method : elimination of internal variables for each module , resolution of the system of equations for junctions variables and regeneration of internal variables of the modules . The post-processing of the code allows a visualisation of the calculation with a colour film.

INFLUENCE OF CONVECTION IN THE MELT ON CELLS AND DENDRITES

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Experimental data are analyzed by the light of recent enlightning diffusive theories for the selection of morphological patterns (cells or dendrites) during directional solidification of binary alloys. It is shown that a new unifying formulation of the dispersed available values for the shape parameters (periodicity λ, radius R_0 and solute concentration C_0 at the tip) naturally appears, which depends only on the segregation coefficient k of solute.

Convenient nondimensional structure parameters can be defined which are only functions of k and ratios of the characteristic lengths: thermal to solutal l_T/l_S, which roughly corresponds to the distance to the threshold, and capillary to thermal l_C/l_T, which is A/k where A is Sekerka's parameter. Moreover λ/R_0, which is geometrically related to a hidden relative width by analogy with Saffman - Taylor fingers,stems as a key for a 2D representation.

Fluid flow in the melt results in a shift from the diffusive branch. By introducing the concept of a diffusive boundary layer ahead of the front, which can be characterized either by its thickness Δ or by the effective partition coefficient k_{eff}, it is shown how the shape parameters are affected by convection.

DYNAMIQUE DES SINGULARITES DANS LES MILIEUX ORDONNES : L'EXEMPLE DES CRISTAUX LIQUIDES .

Yves BOULIGAND (*)

RESUME : Il s'agit de la présentation d'un film réalisé il y quelques années par Jean PEINLEVE sur mes préparations de cristaux liquides. Après un rappel sur les cristaux solides, les principaux types de cristaux liquides sont définis : nématiques, smectiques et cholestériques. L'atat nématique est examiné au microscope polarisant, ainsi que la transition avec l'état liquide ordinaire. Les fluctuations thermiques sont bien visibles au sein du liquide. Les singularités y sont présentes sous forme de fils épais ou fins, qui donnent leur nom à ces cristaux liquides. L'Etat cholestérique ou état nématique helicoïdal est la forme adoptée par les corps nématiques en présence de certaines dissymétries moléculaires. Les singularités des nématiques y forment alors des réseaux dont on peut observer la dynamique en diverses situations expérimentales. Une partie du film est consacrée à des structures biologiques très analogues, observées dans divers tissus squelettiques qui sont en partie constitués de cristaux liquides stabilisés. Las structure et les symétries sont conservés et non la fluidité. On y observe également des singularités étudiables par la méthode des coupes en histologie. La carapace des crabes, une fois décalcifiée, est le matériel idéal pour ce travail.

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RESULTATS MATHEMATIQUES RECENTS MOTIVES PAR LES CRISTAUX LIQUIDES

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Colloque: Equations aux dérivées partielles et modèles continus de transitions de phase , Nice, 18-22 January 1988.

MULTICOMPONENT SOLITONS IN ELASTIC CRYSTALS

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Abstract

In this brief overview multicomponent soliton solutions are exhibited analytically and/or numerically for several classes of elastic crystals that are subject to phase transitions (elastic crystals with a rotatory-like microstructure, electrically poled crystallites, elastic crystals with coupled transverse-longitudinal displacements, diatomic lattices). All models are discrete to start with, while sometimes a full continuum limit is taken and other times only the envelope of the signal is treated in a continuous framework, fast oscillations satisfying still the discrete system. The attention is focused on the case of systems governed by modified Boussinesq equations yielding modified KdV equations or nonlinear Schrödinger equations . Analogies with nonlinear bi-inductance electric lines (which allow for real-time analog simulations) are given and several phenomena such as recurrence and modulational instabilities are pinpointed. This field of study offers beautiful examples where dispersive hyperbolic systems, nonlinear systems of evolution and parabolic equations occur all simultaneously.

ANALOGIES ENTRE DIGITATIONS VISQUEUSES ET CROISSANCE DENDRITIQUE

Yves COUDER (*)

When, between two narrowly spaced glass plates, a gas forces a fluid of larger viscosity to recede their interface is unstable. This Saffman-Taylor instability gives rise to large penetrating air fingers.

We present a series of experiments demonstrating that a localized perturbation of tip of these fingers changes completely their regime of growth so that they become close analogues of cristalline dendrites. This disturbance simulates the role of the surface tension anisotropy which exists in the case of crystal growth and demonstrates the singular role of the finger tip region in the shape selection.

Furthermore we obtain a system which lends itself to a quantitative analysis of the destabilization given rise to the lateral side branching. Using local perturbation and periodic forcing we thus characterize experimentally the instability of a moving curved interface.

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DESCRIPTION A LA LANDAU-GINZBURG DES TRANSITIONS DANS LES SYTEMES HORS-EQUILIBRE

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MODELISATION MACROSCOPIQUE DES MATERIAUX A MEMOIRE DE FORME

Michel FREMOND(*)

On donne un modèle thermodynamique macroscopique des matériaux à mémoire de forme utilisant des variables macroscopiques : température, déformations et proportions des différentes phases. Le modèle rend compte des phénomènes expérimentaux usuels, en particulier de l'éducation du matériau à mémoire de forme.

Un théorème d'existence est démontré dans le cadre de la théorie du second gradient.

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UNSTABLE INTERFACES AND FLUID CHAOS

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The laminar to chaotic transition governs the nature of the fluid mixing process. For laminar flow, mixing is governed by molecular events while in the chaotic regime, fluid modes dominate entrainment and mixing.

Commonly studied mechanisms for interface instability are acceleration, impulsive forces, shear and pressure gradients. In all unstable cases a chaotic regime results in the long time behavior.

Characterization of large time behavior is complicated by a dynamical change of length scales which results from a tendency towards self-organization in such flows.

Scientific progress can be achieved on several levels: the study of one or a small number of interacting modes and the statistical consideration of a large ensemble of modes.

Recent progress of the author and coworkers will be reviewed.

Snowflakes and Spherulites: Moving Boundary Problems With and Without Anisotropy

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From a physicist's perspective, I discuss the basic physics underlying the generation of patterns by diffusion-controlled interface motion. I review numerical and theoretical results obtained on simple models of dendritic growth, and show that crystalline anisotropy is a crucial parameter which must be included in the analysis. As the undercooling and anisotropy strength are varied, different morphologies are generated by the solidification front. These results are obtained from a hydrodynamic analoque of solidification – the Hele-Shaw cell. One of the patterns generated by the analogue, the so-called dense branching morphology, has a counterpart in crystal growth, namely spherulitic growth. I discuss this example, and point out a variety of unsolved problems in this area.

Partial Differential Equations and Continuum Models of Phase Transitions

Nice, January 18-22, 1988

CONTINUUM MODELS OF PHASE TRANSITIONS FOR NON-ISOTHERMAL LIQUID-VAPOR INTERFACES

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The second gradient theory [5], conceptually simpler than that of Laplace, has led to a capillay theory for isothermal liquid-vapor interfaces. The internal energy or the free energy of such fluids is a function of the entropy or temperature, density and gradient of density [2,7,14,15]. Using a three-dimensional continuous medium, one can construct isothermal interfaces connecting a liquid and its vapor [10,17]. In the case of isothermal equilibrium, these fluids are identified with models related to molecular theories [2,9,11,12]. In fact, such media do not allow interfacial layers with large gradients of temperature. For a fluid whose internal energy ε is a function not only of density ρ and specific entropy s but also of gradients of ρ and s, we can imagine non-isothermal equilibrium in zones with large density gradients. We call such a medium a *thermocapillary fluid* [3]. Internal specific energy is written $\varepsilon = f(\rho, s, \operatorname{grad}\rho, \operatorname{grad} s)$. The equation of motion is

$$\rho\Gamma_i = \sigma^j_{i,j} + \sigma^{\ j}_{v\ i,j} - \rho\Omega_{,i}$$

Here, Γ denotes the acceleration vector, Ω the extraneous force potential and σ is the generalization of the stress tensor:

$$\sigma^j_i = -(p - \rho\operatorname{div}\Phi)\,\delta^j_i - \Phi^j\rho_{,i} - \Psi^j s_{,i}$$

where $p = \rho^2\varepsilon_{,\rho}$, $\Phi^i = \rho\,\varepsilon_{,\rho_{,i}}$, $\Psi^i = \rho\,\varepsilon_{,s_{,i}}$

and σ_v denotes the viscous stress tensor.

(The theory yields two new vectors Φ and Ψ [3,4]).

The conservation of energy yields:

$$\frac{\partial e}{\partial t} + \operatorname{div}[(e - \sigma - \sigma_v)V] - \operatorname{div} U + \operatorname{div} q - r - \rho\frac{\partial\Omega}{\partial t} = 0,$$

where $e = \rho(\frac{1}{2}V^2 + \varepsilon + \Omega)$, q is the heat flux vector, r the heat source term and $U = \dot{\rho}\,\Phi + \dot{s}\,\Psi$.
The vector U has the same dimension as does the heat flux vector. It exists even for inviscid fluids.

Let $\theta = \Theta - \frac{1}{\rho}\operatorname{div}\Psi$ where Θ corresponds to the *Kelvin temperature* for compressible fluids. We call θ the *thermocapillary temperature*.
The *generalized Clausius-Duhem inequality* yields:

$$\rho\dot{s} + \operatorname{div}\left(\frac{q}{\theta}\right) - \frac{r}{\theta} \geqslant 0$$

The equations of motion and energy of an inviscid fluid with infinite thermal conductivity imply:

$$\rho(\Theta - \theta_o) - \operatorname{div}\Psi = 0$$

where θ_o is a constant along the motion. The thermocapillary temperature of the fluid is a constant. This equation points out that a thermocapillary fluid may be in non-isothermal equilibrium depending on the temperature $\Theta = \varepsilon_{,s}$.

We draw an inference from this relation, particularly giving an interpretation of the film boiling phenomena [4].
More accurate models of dynamic changes of phase are fluids of grade n [8]. Perfect fluids of grade n (n is any integer) are continuous media with an internal energy per unit mass which is a function of specific entropy, density and spatial gradients up to n-1 order [16]:

$$\varepsilon = \varepsilon(s, \operatorname{grad} s, \ldots, (\operatorname{grad})^{n-1} s, \rho, \operatorname{grad} \rho, \ldots, (\operatorname{grad})^{n-1} \rho)$$

The equation of motion for such perfect fluids has a similar form to the one given by J. Serrin in the case of conservative compressible fluids [8,13]:

$$\Gamma = \theta \operatorname{grad} s - \operatorname{grad}(h + \Omega) \qquad (+)$$

and

$$\dot{s} = 0$$

$$\begin{cases} \theta = \varepsilon_{,s} + \dfrac{1}{\rho} \displaystyle\sum_{m=1}^{n-1} (-1)^m \operatorname{div}_m \left(\rho \frac{\partial \varepsilon}{\partial (\operatorname{grad})^m s} \right) \\ p = \rho^2 \varepsilon_{,\rho} + \rho \displaystyle\sum_{m=1}^{n-1} (-1)^m \operatorname{div}_m \left(\rho \frac{\partial \varepsilon}{\partial (\operatorname{grad})^m \rho} \right) \\ h = \varepsilon + \dfrac{p}{\rho} \end{cases}$$

where div_m notes the divergence operator iterated m times over the physical space. In the particular case of a compressible fluid (n=1), p and h are the pressure and the enthalpy per unit mass. By taking into account the successive gradients of density, we improve the model accuracy of capillary layers in liquid-vapor interfaces.
The conservative flows of perfect fluids are only a mathematical model. So it is with fluids of grade n. However, the study of their structure is fundamentally necessary. The conclusions for fluids of grade n are of the same kind as for compressible fluids [1,6,13]. They are deduced from the universal thermodynamic form (+) of the equation of motion.

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Cavitation Waves in van der Waals Fluids

(M. Grinfeld - Heriot-Watt University)

In this work we examine the influence of capillarity on the structure of travelling wave solutions to regularized equations of the flow of van der Waals fluids. In particular, we prove non-uniqueness results by a combination of phase-space analysis and Wazewski's principle. The exhibited solutions can be interpreted as "cavitation waves", i.e. travelling waves in which vapour and liquid phases are separated by near-vacuum.

SOLUTAL CONVECTION DURING SOLIDIFICATION OF BINARY ALLOYS. EFFECT OF SIDEWALLS

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The solutal convective instability of a binary alloy melt is investigated for the case of directional solidification with sidewalls. The solid - liquid interface is assumed to be rigid and planar. The lateral bounding is characterised by the aspect ratio $\beta = LV/D$ where L is the distance between the walls, V the growth rate and D the coefficient of solute diffusion.

The Boussinesq approximation to the Navier-Stokes equations is assumed to hold for the liquid. The lateral boundaries are rigid and insulating to solute. The stability analysis is carried out by using a Tau - Chebyshev spectral method. As a first step the discussion is restricted to the two-dimensional case with exchange of stabilities. After a test on the Rayleigh-Bénard problem in a finite box, for which a good agreement is obtained with the classical results, the code is applied to directional growth.

The critical Rayleigh number $R_S{}^C$ is determined as a function of the aspect ratio and solute segregation coefficient. The stabilizing effect of the lateral confinement is clear. When β increases, there is first a competition between the odd and even modes in order to rule the unstable pattern, which leads to an alternance of parities. Both modes then gradually merge and $R_S{}^C$ tends toward its asymptotic value for the unconfined case as β becomes infinite.

By considering the more usual diagram for directional solidification, i.e. critical solute concentration versus growth velocity, it can be shown, for lead-tin alloys which are the reference in the literature as well as for lead-thallium alloys which are under study in our group, that it would be nearly impossible to suppress solutal convection by changing the crucible diameter only. Nevertheless, microgravity provides an efficient way for hydrodynamic stabilisation.

Computer facilities for this work were given by the Scientific Council of the Centre de Calcul Vectoriel pour la Recherche.

EQUATIONS AUX DERIVEES PARTIELLES ET MODELES CONTINUS DE PHASES
NICE, 18-22 JANVIER 1988.

Proposition de brève communication par C.GUILLOPE and J.C.SAUT
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Title : Existence results for viscoelastic fluids with a differential constitutive law.

Summary : We consider viscoelastic fluids which obey an Oldroyd type constitutive equation with retardation time. For the mixed Cauchy problem with homogeneous Dirichlet conditions, the local existence and the uniqueness of a smooth solution are proven. Then a global existence result of solutions with small data is shown.

Titre : Résultats d'existence pour des fluides viscoélastiques à loi de comportement de type différentiel.

Résumé : Nous considérons des fluides viscoélastiques décrits par une loi de comportement de type Oldroyd avec temps de retard. On montre, pour le problème de Cauchy mixte avec conditions de Dirichlet homogènes, l'existence locale de solutions régulières. Puis l'existence globale de solutions lorsque les données sont petites est démontrée.

Thermodynamics and free-boundary problems for solidification

Morton E. Gurtin

Abstract.

The classical theory of Stefan is far too simplistic to account for the myriad of phenomena that occur during solidification. In this talk I discuss a nonequilibrium thermodynamics for solidification in which the interface between phases possesses a structure of its own. The first two laws are appropriate to a continuum and contain interfacial contributions for both energy and entropy. The constitutive equations for the interface allow for a dependence on the orientation and on the normal velocity of the interface. Free-boundary problems, for the growth of a crystal in a liquid melt, are derived and discussed.

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YOUNG'S MEASURES AND THE MICRO-STRUCTURE OF CRYSTALS (II).

RICHARD JAMES
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PHASE TRANSISTIONS IN CRYSTALS : THEORY AND ANALYSIS

David KINDERLEHRER(*)

ABSTRACT : A brief summary of the approach from continuum theory to phase transitions and variant structures in crystals followed by some applications. Some examples are (a) the analysis of Young measures , (b) the role of internal variables, and (c) implications for computation.

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SINGULARITIES OF THE ORDER PARAMETER IN CONDENSED MATTER PHYSICS

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The order parameter is a thermodynamic variable, conjugated to a field, which describes at a macroscopic scale specific properties of condensed matter deriving from microscopic interactions (elasticity deriving from atomic correlations ; ferro, antiferromagnetism and spin glass behaviour deriving from spin interactions ; superfluidity deriving from Bose condensation, etc...). The order parameter, whatever its tensorial nature might be, is characterized by an amplitude, which varies with temperature T, and a generalized phase, whose spatial variations at given T describe the distortions of the medium.

At the temperature of transition between two thermodynamic phases (here we use the term phase in its usual sense of ~~the~~ thermodynamic state), the amplitude of the order parameter vanishes. The low temperature phase shows up broken symmetry with respect to the high temperature one, in second order phase transitions and in many other cases.

The singularities of the distortion of the medium at some fixed temperature are related to the topological properties of the phase of the order parameter, and can be classified accordingly.

In this lecture, we shall develop mostly the last topic, and will extend the notion of singularity of the order parameter to disordered and aperiodic systems.

Fingering in porous media: Stochastic models and physical simulators.

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ABSTRACT

Immiscible displacements in porous media with both capillary and viscous effects can be characterized by two dimensionless numbers, the capillary number C which is the ratio of viscous forces to capillary forces and the ratio M of the two viscosities. For certain values of these numbers, either viscous or capillary forces dominate and displacement takes one of the basic forms: (a) viscous fingering, (b) capillary fingering or (c) stable displacement. The first part of this study presents the results of network simulators (100×100 and 25×25 pores) based on the physical rules of the displacement at the pore scale. The second part describes a series of experiments performed in transparent etched networks. Both the computer simulations and the experiments cover a range of several decades in C and M. They clearly show the existence of the three basic domains (capillary fingering, viscous fingering and stable displacement) within which the patterns remain unchanged. The domains of validity of the three different basic mechanisms are mapped onto the plane with axes C and M, and this mapping represents the "phase-diagram" for drainage. In the final section we present three statistical models (percolation, diffusion-limited aggregation and anti-DLA) which can be used for describing the three "basic" domains of the phase-diagram.

Computational Results for Defects and Transitions in Ordered Materials

Mitchell Luskin
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Computational results will be presented for defects in liquid crystals. The instability of defects with degree greater than one will be demonstrated. Results for the interaction of electric and magnetic fields with liquid crystals will also be presented.

ASYMPTOTIC VARIATIONAL PROBLEMS IN THE GRADIENT THEORY OF PHASE TRANSITIONS

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Swelling and Collapse in Polyelectrolytic Gels

by
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Abstract

Swelling and collapse of a polyelelectrolyte are shown to be the result of a delicate balance of three "thermodynamic forces", viz. osmotic pressure, polymer elasticity, and van der Waals attraction. The behaviour of the polyelectrolyte gel is studied under isotropic pressure and under uniaxial and biaxial loading.

Front migration in phase separation

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Abstract

I will discuss the generation and evolution of interfacial layers in some singularly perturbed nonlinear partial differential equations which model phase separation. In particular, the Cahn-Hilliard equation and the bistable reaction diffusion with stable states of equal energy will be considered. In these systems, fronts migrate slowly in order to lower the energy of the system concentrated in interfacial layers. Some rigorous and some formal results describing the evolution and stability of evolving interfaces will be described.

STABILITE DE FRONTS COURBES

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INSTABILITIES IN SHEAR FLOWS OF VISCO-ELASTIC FLUIDS WITH FADING MEMORY.

Bradley PLOHR (*)

Abstract : For certain models of visco-elastic fluids with fading memory, classical steady Poiseuille flox does not exist beyond a maximal wall shar stress ; traditionally this was regarded as a defect. We report on work that shows these models to correctly describe experimental observation of "spurt" phenomena. The flow equations, which are of mixed type, may be cast in the form of gas dynamics, and the transition to spurt flow is analogous to a dynamically generated phase transition.

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EQUATIONS AUX DERIVEES PARTIELLES ET MODELES CONTINUS DE TRANSITIONS DE PHASES

Nice, 18-22 Janvier 1988

Buoyancy and Thermocapillary Convection in melts during unidirectional crystallization (horizontal Bridgman)

by

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ABSTRACT

During unidirectional solidification from their melts , like in horizontal Bridgman boats, crystals are directly influenced by the convective motions occurring in these melts . Due to the crystal pulling and the rejection of one of the species (dopants or impurities) a concentration layer is formed in the front of the crystallization interface . In most of the applications for metallic crystals or semi-conductors , the kinematic viscosity is very large compared to the diffusion coefficient (large Schmidt number , Sc=v/D) . Thus an even small flow motion can strongly influence the concentration field and induce a so-called radial segregation (concentration gradient in the direction parallel to the solidification interface).

The convective motions in the melt can be driven by the buoyancy and , for open boat systems, by the surface stresses induced by the variation of the surface tension along the upper free-surface (the so-called thermocapillary effect) .

We present results concerning a direct simulation of the radial segregation in the case of a 2D model, for a steady-state solution . The model takes into account the 2D deformation of the crystal-melt interface , but the pulling velocity is assumed to be small enough in order to avoid morphological instabilities . We also present results showing the transition to oscillatory regimes in low Prandtl-number fluids in rectangular cavities , with the crystallization interface motion neglected (zero pulling velocity). This numerical study is complemented with a more theoretical approach , by applying (hydrodynamic) stability theory to a base one-dimensional , "recirculating" , flow valid for infinitely long (horizontal) cavities . Thus, the role of the (thermal and dynamical) boundary conditions applied to the upper surface of the melt has been emphasized .

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Change of type and short waves instabilities in the flow of visco-elasticité fluids

J. Claude SAUT

Les modèles usuels de fluides viscoélastiques conduisent à des systèmes d'équations aux dérivées partielles complexes;on y rencontre des phénomènes tels que perte d'évolution,changement de type,...rendant leur étude difficile.

Le but de l'exposé est de présenter les résultats connus ainsi que les nombreuses questions ouvertes.

INITIAL CONDITIONS IMPLIED BY $t^{1/2}$ SOLIDIFICATION OF A SPHERE WITH CAPILLARITY AND INTERFACIAL KINETICS

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ABSTRACT

We explore the initial conditions implied by $t^{1/2}$ growth of a spherical crystal solidifying from a pure, undercooled melt, including the effects of both capillarity and interface kinetics, and relate our findings to initial conditions that would be expected on the basis of classical nucleation theory. For crystal sizes near the nucleation radius, the calculated temperature profiles show a cold region ahead of the advancing interface that is even more undercooled than the undercooled bath. This cold region acts as a local heat sink that compensates for the reduced growth speed that would otherwise result from capillarity and kinetics, leading to precisely the same $t^{1/2}$ growth law that would have been obtained had both capillarity and kinetics been neglected. We submit that this $r^{1/2}$ solution should not be taken seriously in the context of the classical theory of nucleation of a crystal from an isothermal melt. Furthermore, we estimate that use of this solution in a morphological stability analysis would lead to an onset of instability at smaller crystal sizes than predicted by the classical analysis.

*This work is in cooperation with P. W. Voorhees, S. R. Coriell, and G. B. McFadden of the National Bureau of Standards, Gaithersburg, MD 20899, USA, and will be published in the Journal of Crystal Growth. One of us (RFS) received partial support from the National Science Foundation under Grant DMR84-09397.

NON STRICTLY HYPERBOLIC CONSERVATION LAWS OF OIL RECOVERY

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ABSTRACT : A family of 2×2 systems of conservation laws is presented. These equations model the flow of three fluids in an oil reservoir. Properties of the equations include loss of strict hyperbolicity and the presence of undercompressive shock wave solutions. Loss of strict hyperbolicity is illustrated in several models, and a preliminary analysis of viscous profiles for undercompressive shocks is presentated. These shocks are related to dynamic phase transitions associated with non-monotone stress-strain laws in elasticity, and with Van der Waalsgases.

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ABSTRACT

VISCOSITY, CAPILLARITY LIMITS
AND DYNAMICS OF PHASE TRANSITIONS

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This lecture will first review the role of using viscosity and capillarity as an admissibility criteria for propagating phase bounderies in materials exhibiting phase transitions. Then we will move on to see how the same idea can be used to find the solution of the Riemann initial problem for a van der Waals fluid via a variant of the method of vanishing viscosity and capillarity.

EQUATIONS AUX DERIVEES
PARTIELLES ET MODELES CONTINUS
DE TRANSITIONS DE PHASES

(Nice, January 18-22, 1988)

A B S T R A C T

ADIABATIC PHASE CHANGES, FAST AND SLOW
(EXPERIMENTAL RESULTS)

P.A THOMPSON (U S A)

Many phase changes proceed spontaneously from a metastable state, without significant heat transfer. Diverse examples are given, including cloud formation, the synthesis of diamond and certain natural catastrophes.

New discoveries in the field of very rapid phase changes in polyatomic (retrograde) fluids are discussed. These include very rapid shock-like, liquid-vapour transitions. Some of these phenomena have been traditionally regarded as impossible. Chapman-Jouget states are encountered in at least three separate discontinuities with phase change. Near-critical states achieved by means of shock waves show remarkable behavior, including very small soundspeeds.

A number of puzzles remain: The stability or instability of the shockfronts, the relation of microscopic ring vortices to homogenous nucleation, the structure of the rapidly moving pressure and phase discontinuities and the existence of non-equilibrium critical states with infinite compressibility..

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